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Properties of string theory on Kaluza-Klein Melvin background

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ABSTRACT: We investigate some properties of string theory on Kaluza-Klein Melvin background. We discuss what happens when closed string tachyons in the theory condense, by using a linear sigma model of the Kaluza-Klein Melvin background.

KEYWORDS: Superstrings and Heterotic Strings, String Duality, Supersymmetry Breaking.

Contents

1. Introduction

The stability of vacua of string theory is one of the main topics in recent researches. It is well-known that there are infinitely many perturbative vacua in string theory, and some of them contain tachyons in their spectra. The existence of the tachyons has been considered as an indication of instabilities, not of the inconsistency, of the vacuum. However, such vacua have not been discussed, although some attempts were made [1], because of the lack of the knowledge of the mechanism for the stabilization of them.

Now we know the fate of perturbative vacua which contain tachyons in the open string sector [2]. See also recent review papers, e.g. [3] and references therein. The existence of open string tachyons is related to the existence of an unstable D-brane system. The tachyons acquire nonzero vevs, at which the effective potential takes a minimum value, and the fluctuations around such configuration do not contain any tachyon. This process describes the decay of the unstable D-brane system into a stable vacuum or a stable D-brane system.

On the other hand, the stabilization of closed string tachyons is a harder problem to clarify. It would be because, in many cases, the closed string tachyons are not related to any unstable subsystem in the theory. Some interesting discussions have been done on this problem [4]–[9] , which seem to be right directions to understand the closed string tachyons.

In some cases, the following background geometry plays an important role,

$$ds^2 = ds^2_{flat} + dr^2 + r^2(d\theta + qdy)^2 + dy^2 \,. \tag{1.1}$$

This is known as the Kaluza-Klein Melvin background [10, 11]. Its generalizations are discussed in [12]. Here q is a real constant and the y-direction is compactified on S^1 with the radius R. This metric is locally flat, so this would be an exact background of string theory and M-theory. All the supersymmetries are broken in this background since the spin structure is changed by turning on q.

The eleven-dimensional KK-Melvin can be interpreted as a type IIA background, by the dimensional reduction along the y-direction. The type IIA background has a localized R-R flux, known as a fluxbrane [5], and the dilaton diverges as it goes far away from the core of the flux, so this background cannot be analyzed by perturbative calculations. It is conjectured in [4] that type IIA theory on this background is equivalent to type 0A theory. The latter theory contains a closed string tachyon, and it is expected that type 0A theory would decay into type IIA theory on the flat spacetime, via a condensation of the closed string tachyon. Similar arguments have been applied to non-supersymmetric heterotic theories [7]. There are also some investigations on closed string tachyons which appear in different contexts [8, 9].

A common property of the systems mentioned above is that they are superstring theories on supersymmetry-breaking backgrounds. Therefore it would be reasonable that such theories are stabilized to be the theories on supersymmetric vacua. Then one is led to think that the closed string tachyon condensation would deform the background geometry in a drastic way. So investigations of the mechanism of the tachyon condensation would give us many insights into the description of spacetime in string theory.

In this paper, we investigate the properties of superstring theory on the ten-dimensional KK-Melvin in more detail. This system is interesting since closed string tachyons appear in a region of the parameter space of the background. In addition, the fate of them could be deduced; the system would decay into the same string theory on a supersymmetric background. Moreover, the system includes non-supersymmetric theories, i.e. type 0 theories and the non-supersymmetric heterotic theories, as various limits of the system. For simplicity, we will mainly consider type IIA theory on the KK-Melvin, but the results would be applicable to the heterotic cases. In fact, one interesting open question is the relation between the tachyon condensation and the gauge symmetry breaking in the heterotic theories [7]. So discussions for the heterotic cases are also relevant.

The organization of this paper is as follows. We recall the worldsheet action for the KK-Melvin in section 2, and try to understand the pattern of the mass spectrum in the zero-winding sector, in terms of the field theory language, in section 3. We also comment on marginal deformations of the worldsheet theory. Tachyons only appear in the winding sectors, so the similar discussions for the tachyons are difficult. We just specify the parameter region in which tachyons appear, in section 4. In section 5, we construct a linear sigma model of a string on the KK-Melvin. Then we investigate deformations of the background

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due to the tachyon condensation, by using the model, in section 6. This analysis has close similarity to [8]. In section 7, we briefly comment on the tachyon condensation and the gauge symmetry breaking in the heterotic theories.

2. Nonlinear sigma model for the KK-Melvin

The action for type IIA theory on the ten-dimensional KK-Melvin (1.1) is $S = S_{flat} + S_{\text{Melvin}}$ [13], where

$$\begin{aligned} S_{\text{Melvin}} = & -\frac{1}{4\pi\alpha'}\int d^2\sigma\left\{(\partial_a X^\dagger - iq\partial_a Y X^\dagger)(\partial_b X + iq\partial_b Y X) + \partial_a Y \partial_b Y\right\}\eta^{ab} + \\ & + \frac{i}{\pi}\int d^2\sigma\Big\{\psi_L^\dagger(\partial_- + iq\partial_- Y)\psi_L + \psi_R^\dagger(\partial_+ + iq\partial_+ Y)\psi_R + \\ & \qquad + \psi_L^Y\partial_-\psi_L^Y + \psi_R^Y\partial_+\psi_R^Y\Big\}, \end{aligned} \tag{2.1}$$

and S_{flat} is the action for the free theory corresponding to the flat spacetime directions x^μ with $\mu = 0, \ldots, 6$. We have employed the RNS formalism. The fields X, $\psi_{L,R}$ are defined as follows,

$$\begin{aligned} X &= X^7 + iX^8\,, \\ \psi_{L,R} &= \psi^7_{L,R} + i\psi^8_{L,R}\,. \end{aligned}$$

Note that the vector indices of the fermions are the Lorentz indices, not the Einstein ones [13]. The y-direction is compactified on S^1, so the field Y is a compact boson, satisfying

$$Y(\sigma + 2\pi) = Y(\sigma) + 2\pi w R\,, \tag{2.2}$$

where w is the winding number.

The action (2.1) can be reduced to the free action by the following field redefinitions,

$$\begin{aligned} X &= e^{-iqY}\tilde{X}\,, \\ \psi_{L,R} &= e^{-iqY}\tilde{\psi}_{L,R}\,. \end{aligned}$$

Then all the fields satisfy the free wave equations. So the theory is described by the following free twisted fields,

$$\begin{aligned} \tilde{X}(\sigma + 2\pi) &= e^{2\pi i wqR}\tilde{X}(\sigma)\,, \\ \tilde{\psi}_{L,R}(\sigma + 2\pi) &= \pm e^{2\pi i wqR}\tilde{\psi}_{L,R}(\sigma)\,, \end{aligned}$$

and free untwisted fields corresponding to the flat spacetime directions and the y-direction.

The only difference from just a free theory is a zeromode p of the field Y. Its canonical momentum P_Y is

$$\begin{aligned} P_Y &\equiv \frac{\delta S}{\delta\partial_\tau Y} \\ &= \frac{1}{2\pi\alpha'}\partial_\tau Y + \frac{iq}{4\pi\alpha'}\left\{X(\partial_\tau X^\dagger - iq\partial_\tau Y X^\dagger) - X^\dagger(\partial_\tau X + iq\partial_\tau Y X)\right\} - \\ &\quad - \frac{q}{2\pi}\left(\psi_L^\dagger\psi_L + \psi_R^\dagger\psi_R\right). \end{aligned} \tag{2.3}$$

It is the zeromode of P_Y that is quantized, so the zeromode p is

$$\begin{aligned} p &\equiv \int_0^{2\pi} d\sigma \frac{1}{2\pi\alpha'}\partial_\tau Y \\ &= \frac{m}{R} - qJ\,, \end{aligned} \tag{2.4}$$

where m is an integer, and J is the angular momentum operator in the 7-8 plane,

$$\begin{aligned} J = \int_0^{2\pi} d\sigma \Big[&\frac{i}{4\pi\alpha'}\Big\{X(\partial_\tau X^\dagger - iq\partial_\tau Y X^\dagger) - X^\dagger(\partial_\tau X + iq\partial_\tau Y X)\Big\} - \\ &- \frac{1}{2\pi}(\psi_L^\dagger\psi_L + \psi_R^\dagger\psi_R)\Big]\,. \end{aligned} \tag{2.5}$$

3. Zero-winding sector

3.1 Mass spectrum

Consider first the zero-winding sector. In this sector, $\tilde{X}$ and $\tilde{\psi}_{L,R}$ have the ordinary periodicities. So the effect of the nontrivial background appears only in the zeromode p. The mass operator and the level-matching condition are as follows,

$$\alpha' M^2 = \alpha'\left(\frac{m}{R} - qJ\right)^2 + 2(N + \tilde{N} - a)\,, \tag{3.1}$$

$$N = \tilde{N}\,, \tag{3.2}$$

where N, $\tilde{N}$ are the ordinary level operators for the left- and the right-movers, respectively, and a is a normal ordering constant.

One can easily see, from the expression (3.1), that there is no tachyonic mode in the zero-winding sector. In fact, as will be shown in section 4, tachyons appear in the winding sectors. The mass of the states in the zero-winding sector are shifted according to their eigenvalues of J.

Let us focus on some states in the NS-NS sector, which are massless when $q = 0$. They are the graviton G_{MN}, the anti-symmetric tensor B_{MN} and the dilaton Φ. The followings are the eigenvalues of J for the states in the NS-NS sector, corresponding to the components of the above fields,

$$\begin{aligned} J = 2 &: G_{zz},\ B_{zz}, \\ 1 &: G_{\mu z},\ G_{yz},\ B_{\mu z},\ B_{yz}, \\ 0 &: G_{\mu\nu},\ G_{\mu y},\ G_{yy},\ G_{z\bar{z}}, \\ &\quad B_{\mu\nu},\ B_{\mu y},\ \Phi, \\ -1 &: G_{\mu\bar{z}},\ G_{y\bar{z}},\ B_{\mu\bar{z}},\ B_{y\bar{z}}, \\ -2 &: G_{\bar{z}\bar{z}},\ B_{\bar{z}\bar{z}}, \end{aligned} \tag{3.3}$$

where $G_{\mu z} = (G_{\mu 7} - iG_{\mu 8})/2$, etc. For a small value of q, massless fields in the zero-winding sector are the components of the above fields whose eigenvalues of J are zero, and the other components become massive with the masses of order q.

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It is expected that properties of the states in the zero-winding sector can be understood in terms of the field theory language. Since the above results are obtained from the tree-level analysis, the mass shift discussed above will be seen by investigating the corresponding classical supergravity lagrangian. In fact, one needs only kinetic terms of the spacetime fields to see the mass shift.

Consider a vector field V^A, for simplicity. Here A is the Lorentz index. Since the massless fields correspond to the states created by the fermionic oscillators, their indices should be the Lorentz ones, as noted in the previous section. The kinetic term of V^A is

$$G^{MN}\nabla_M V^A \nabla_N V_A = G^{MN}\left(\partial_M V^A + \omega_M{}^A{}_C V^C\right)\left(\partial_N V_A + \omega_{N,AD}V^D\right), \tag{3.4}$$

where $\omega_M{}^A{}_B$ is the spin connection. For the metric (1.1), most of the components of the spin connection vanish, except for the followings,

$$\omega_y{}^{78} = -\omega_y{}^{87} = -q\,. \tag{3.5}$$

Then $\nabla_{\mu,7,8}$ are reduced to the ordinary partial derivatives. The kinetic term (3.4) becomes

$$\eta^{\mu\nu}\partial_\mu V^A \partial_\nu V_B + \left(\partial_7 V^A\right)^2 + \left(\partial_8 V^A\right)^2 + \left(\nabla_y V^A - q\left(x^7\partial_8 - x^8\partial_7\right)V^A\right)^2. \tag{3.6}$$

The last term can be rewritten as

$$(\partial_y V^\mu - iqLV^\mu)^2 + |\partial_y V^z + iqV^z - iqLV^z|^2 + (\partial_y V^y - iqLV^y)^2, \tag{3.7}$$

where $L = -i(x^7\partial_8 - x^8\partial_7)$ is the orbital angular momentum, and $V^z = V^7 + iV^8$. If the vector field V^A is a simultaneous eigenstate of ∂_y and L, then the terms (3.7) become mass terms with the masses

$$m^2 = \begin{cases} \left(\frac{k}{R} - q(l-1)\right)^2 & \text{for } V^z\,, \\ \left(\frac{k'}{R} - ql\right)^2 & \text{for } V^\mu,\ V^y\,. \end{cases} \tag{3.8}$$

where l is an eigenvalue of L. The corresponding result for an arbitrary bosonic field can be obtained in the same way. For example, a field ϕ with n_z z-indices and $n_{\bar z}$ $\bar z$-indices has a mass

$$m^2 = \left(\frac{k}{R} - q(l - (n_z - n_{\bar z}))\right)^2. \tag{3.9}$$

This value of m^2 coincides with the value of p^2 in the string spectrum. Note that the total angular momentum j for the field ϕ should be $l + (n_z - n_{\bar z})$. This is not inconsistent with the result (3.9). The operator (3.1) acts on the states, and a state with z-index corresponds to a field with $\bar z$-index and vice versa.

The situation is the same for spinor fields. Spinor states have half-integral eigenvalues of J, so there is no massless fermion unless qR is an even integer. In particular, at $qR = 1$ all spinor states have masses of order at least $1/R$, while some massless bosons appear and the translational invariance is restored. So all spacetime fermions are infinitely massive in the $R \to 0$ limit, which corresponds to the type 0A limit [4, 14].

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As mentioned before, the KK-Melvin breaks all the supersymmetries. The restoration of the supersymmetries at $q = 0$ can be easily understood; some of massive fermions (and also bosons) just become massless, and degrees of freedom of bosons and fermions become the same. Since type 0A theory is expected to be dual to type IIA theory on the KK-Melvin in the $R \to 0$ limit with holding $qR = 1$, the restoration of supersymmetry in this theory after the tachyon condensation would not be a difficult problem, in view of the dual type IIA picture; infinitely massive fermions become massless. This would suggest that, in type 0A theory, some nonperturbative fermionic states become massless after the tachyon condensation. It is very interesting to clarify the mechanism for the restoration of supersymmetry in view of type 0A theory.

3.2 Marginal deformations of the worldsheet theory

In the CFT viewpoint, the tachyon condensation is described by a relevant deformation which corresponds to an off-shel static configuration of the tachyon. As discussed in [4, 11], a theory on the KK-Melvin would decay into the theory on the flat spacetime. So, naively, the tachyon condensation seems to be described by a marginal deformation which modifies only the background field configurations, contrary to the above expectation. In this section, we discuss some marginal deformations of type IIA theory on the KK-Melvin.

The condensation of a massless state corresponds to a marginal deformation of the theory. We focus on the condensation of the gravitons. The massless components of the metric for generic q are

$$G_{\mu\nu}\,, \qquad G_{\mu y}\,, \qquad G_{yy}\,, \qquad G_{z\bar{z}}. \tag{3.10}$$

The last two would be important for the deformations of the KK-Melvin.

The condensation of the state corresponding to G_{yy} changes the coefficient of $(\partial Y)^2$ in S_{Melvin}. Denote the coefficient as C. Then, by rescaling Y as

$$Y = \frac{1}{\sqrt{C}} Y'\,, \tag{3.11}$$

the action becomes again the same form as S_{Melvin}, with different parameters q' and R',

$$q' = \frac{q}{\sqrt{C}}\,, \qquad R' = \sqrt{C} R\,. \tag{3.12}$$

Therefore, this deformation changes the radius R while keeping qR fixed. This seems natural since the combination qR is related to the spin structure of the background geometry.

The deformation corresponding to $G_{z\bar{z}}$ is in fact equivalent to that corresponding to G_{yy}. Remember that the indices of the spacetime fields are the Lorentz indices. So the condensation would change the coefficient of $|\partial\tilde{X}|^2$, not $|\partial X|^2$, in S_{Melvin}. Thus this is equivalent to the rescaling of Y.

Interestingly, as shown above, marginal deformations of the KK-Melvin change the parameters in a restricted manner. In particular, changing only q is not marginal. This seems consistent with the fact that pair creations of D-branes, which is certainly not an on-shell phenomenon, decrease the value of q [4, 11]. In section 6, we will discuss the tachyon condensation on the KK-Melvin. It will be shown that the phenomenon investigated there does not correspond to the marginal deformations of the worldsheet theory discussed above.

4. Tachyons in the winding sectors

Tachyonic modes appear in the winding sectors. They are only in the NS-NS sector [13]. Since the boundary condition of $\tilde{X}$ is twisted, the tachyons do not have zeromodes for the 7,8-directions. This can be understood in terms of the geometry of the KK-Melvin. One can see that the S^1 in the KK-Melvin is twisted, that is, the translation along the S^1 is accompanied by a rotation in the 7-8 plane. So winding strings are also winding around the origin of the 7-8 plane. Thus the winding states are localized in the 7-8 plane. In general, properties of winding states are stringy, so it would be difficult to understand them in terms of the field theory language. One way to analyze them is to see the T-dual theory, in which tachyons appear in the zero-winding sector [15]. Such arguments for the heterotic theories have been given in [7].

In this section, we will just specify the parameter region in which tachyons appear. The perturbative spectrum of this system is already analyzed in [13].

It can be shown that the investigation for the $w = 1$ secotr is sufficient. The tachyonic region in the parameter space for any $w \neq 1$ sector is included in the region for $w = 1$.

The mass operator and the level-matching condition in the $w = 1$ sector are as follows,

$$\alpha' M^2 = \alpha' p^2 + \frac{(wR)^2}{\alpha'} + 2(N_{\text{free}} + \tilde{N}_{\text{free}} + N_{qR} + \tilde{N}_{qR} + N_Y + \tilde{N}_Y + a)\,, \tag{4.1}$$

$$N_{\text{free}} + N_{qR} + N_Y - \tilde{N}_{\text{free}} - \tilde{N}_{qR} - \tilde{N}_Y = -m + RJ, \tag{4.2}$$

where N_{free}, N_Y etc. are the ordinary level operators for the fields corresponding to the x^μ- and y-directions, respectively. The level operators N_{qR} and $\tilde{N}_{qR}$ for the twisted fields have the following forms,

$$\begin{aligned}
N_{qR} &= \sum_{k\in\mathbf{Z}} :\alpha^\dagger_{n-qR}\alpha_{n-qR}: + \sum_{k\in\mathbf{Z}} \left(n + \frac{1}{2} - qR\right) :b^\dagger_{n+\frac{1}{2}-qR} b_{n+\frac{1}{2}-qR}:\,,\\
\tilde{N}_{qR} &= \sum_{k\in\mathbf{Z}} :\alpha^\dagger_{n+qR}\alpha_{n+qR}: + \sum_{k\in\mathbf{Z}} \left(n + \frac{1}{2} + qR\right) :b^\dagger_{n+\frac{1}{2}+qR} b_{n+\frac{1}{2}+qR}:\,,
\end{aligned}$$

where the mode operators satisfy

$$\begin{aligned}
&[\alpha_{m-qR}, \alpha^\dagger_{n-qR}] = (m - qR)\delta_{mn}\,, \qquad &&[\alpha_{m+qR}, \alpha^\dagger_{n+qR}] = (m + qR)\delta_{mn}\,,\\
&\{b_{m-qR}, b^\dagger_{n-qR}\} = \delta_{mn}, &&\{b_{m+qR}\,, b^\dagger_{n+qR}\} = \delta_{mn}\,,
\end{aligned} \tag{4.3}$$

and a is the normal ordering constant.

The mass spectrum is periodic in qR with the period two. So we restrict the parameter region to $0 \leq qR \leq 2$. In this region, the normal ordering constant a takes the following values,

$$a = \begin{cases} qR - 1\,, & \left(0 < qR < \frac{1}{2}\right)\\ -qR\,, & \left(\frac{1}{2} < qR < 1\right)\\ qR - 2\,, & \left(1 < qR < \frac{3}{2}\right)\\ -qR + 1\,. & \left(\frac{3}{2} < qR < 2\right) \end{cases} \tag{4.4}$$

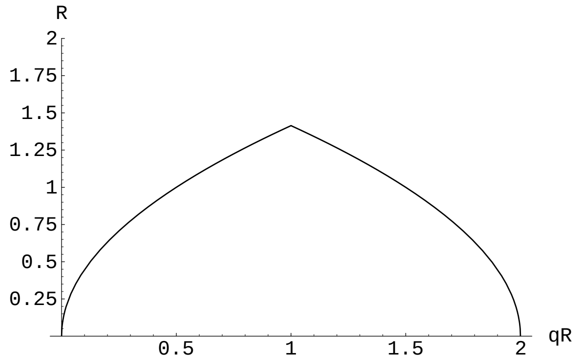

Figure 1: Tachyonic region

Note that this is the only negative contribution to the mass squared. Since the constant a is not less than -1, a state is not tachyonic if the state is created by oscillators of the fields except for $\tilde{X}$, $\tilde{\psi}_{L,R}$.

The lightest state in the spectrum is

$$\begin{aligned} &b^{\dagger}_{\frac{1}{2}-qR} b_{qR-\frac{1}{2}} |m=0\rangle_{NS\text{-}NS}\,, \qquad \left(0 < qR < \frac{1}{2}\right) \\ &b_{\frac{3}{2}-qR} b^{\dagger}_{qR-\frac{3}{2}} |m=0\rangle_{NS\text{-}NS}\,, \qquad \left(\frac{3}{2} < qR < 2\right) \end{aligned} \tag{4.5}$$

and $|m=0\rangle_{NS\text{-}NS}$ for the other region, in which the GSO-projection is reversed. If the above state has a non-negative mass squared, then the theory is tachyon-free. The line on which the lightest state becomes massless is determined as follows,

$$\begin{cases} R^2 = 2\alpha' qR\,, & (0 < qR < 1) \\ R^2 = 2\alpha'(2 - qR)\,. & (1 < qR < 2) \end{cases} \tag{4.6}$$

This is shown in figure 1. When the parameters of the KK-Melvin correspond to a point under this line, there exist tachyonic modes in the winding sectors. Note that the point $(R, qR) = (0, 1)$ corresponds to the type 0A limit.

Each point in figure 1 represents a string theory on a KK-Melvin. If one chooses a point in the tachyonic region, the corresponding theory is unstable, and it would decay into some stable theory without any tachyon. It is conjectured in [4, 5] that the endpoint of the decay would be a supersymmetric vacuum which corresponds to a point on the line $qR = 0$ or equivalently $qR = 2$. In the next section, we will discuss which point is the endpoint of the decay, in more detail. In particular, we will focus on the change of R through the decay.

5. Linear sigma model

In this section, we would like to discuss the closed string tachyon condensation in string theory on the KK-Melvin. An interesting approach is investigated in [8] toward the understanding of a similar phenomenon. They analyze, by using a D-brane probe, type II theory compactified on non-compact orbifolds in which tachyons appear in the twisted sectors. And they conjecture that the condensation of the tachyons deforms the background geometry, and finally the background becomes a supersymmetric one. It seems natural that the tachyon condensation changes the background spacetime since both tachyons and gravitons are closed string states. Therefore, to analyze such phenomena, one should have to employ a formalism in which the dependence on the background geometry is implicit.

We will use a linear sigma model [16] to investigate string theory on the KK-Melvin. The model is not related to any D-brane, so the result we will obtain below would be applicable to the heterotic theories. The structure of the model is very similar to the worldvolume theory of the probe D-brane discussed in [8], so the similar computations can be applied to our model.

5.1 KK-Melvin as a quotient space

It is convenient for our purpose to describe the KK-Melvin in a slightly different way. We consider the case with $qR = k + 1/n$, where k is an integer and n is a positive integer. Then the corresponding KK-Melvin can be obtained as a $\mathbf{Z}_n$ quotient space.

Recall that the KK-Melvin can be obtained from the flat spacetime,

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + r^2 d\tilde{\theta} + dy^2 \,, \tag{5.1}$$

by the following identifications,

$$y \sim y + 2\pi m R, \tag{5.2}$$

$$\tilde{\theta} \sim \tilde{\theta} + 2\pi l + 2\pi m \frac{1}{n} \,, \tag{5.3}$$

where m, l are integers.

Every integer m can be written uniquely as

$$m = rn + s \,, \qquad (r, s \in \mathbf{Z} \,, \quad 0 \le s < n). \tag{5.4}$$

So the identifications (5.2)(5.3) are rewritten as follows,

$$\begin{aligned} y &\sim y + 2\pi r n R + 2\pi s R \,, \\ \tilde{\theta} &\sim \tilde{\theta} + 2\pi l + 2\pi r + 2\pi \frac{s}{n} \,. \end{aligned}$$

When $s = 0$, these identifications are nothing but the S^1 compactification with the radius nR. Therefore the KK-Melvin can be expressed as

$$\mathbf{R}^7 \times \left(\mathbf{R}^2 \times S^1 \right) / \mathbf{Z}_n \,. \tag{5.5}$$

The action of an element $\sigma_s \in \mathbf{Z}_n$ is

$$\sigma_s : \begin{cases} y \mapsto y + 2\pi s R \,, \\ \tilde{\theta} \mapsto \tilde{\theta} + 2\pi s / n \,. \end{cases} \tag{5.6}$$

This expression is useful to construct a linear sigma model of the KK-Melvin, which will be shown below.

5.2 A model

We describe a linear sigma model of the KK-Melvin in this subsection. The model for the flat spacetime directions is trivial, so we only consider the three-dimensional part corresponding to the worldsheet action S_{Melvin}. We will restrict ourselves to considering bosonic degrees of freedom in the KK-Melvin.

It should be noted that the worldsheet $N = 2$ supersymmetry is needed to ensure the validity of the following analyses. In fact, the metric of the moduli space discussed below is modified by the renormalization. It would be desirable to extend the linear sigma model below to a supersymmetric one.

The linear sigma model we analyze below is a two-dimensional gauge theory. The gauge group is $\mathrm{U}(1)^{n+1}$, whose gauge fields are A^i ($i = 0, \ldots, n-1$) and A. There are also $n+2$ scalar fields $z_0, \ldots, z_{n-1}$ and w_1, w_2. They are charged under the gauge fields. The scalar z_i has the charge $+1$ for A^i and -1 for A^{i+1} (the indices have to be considered modulo n), and both scalars w_1, w_2 have the charge $+1$ for A. Note that one linear combination of A^i decouples. We choose the potential as follows,

$$V = \sum_{i=0}^{n-2} \left(|z_i|^2 - |z_{i+1}|^2\right)^2 + \left(|z_{n-1}|^2 - |z_0|^2\right)^2 + \left(|w_1|^2 - \zeta^2\right)^2 + \left(|w_2|^2 - \zeta^2\right)^2 , \tag{5.7}$$

where $\zeta > 0$, in order to obtain the desired theory in the IR limit.

In the IR limit, this theory flows into a nonlinear sigma model whose target space is the moduli space $\mathcal{M}$ of the theory. The moduli space is the space of vevs of the scalar fields which minimize the potential, modded out by the gauge symmetry. The potential has minima at

$$\begin{aligned} |z_0| &= \cdots = |z_{n-1}| , \\ |w_1| &= |w_2| = \zeta . \end{aligned}$$

We fix the gauge symmetry by the following conditions,

$$\arg(z_0) = \cdots = \arg(z_{n-1}) = \arg(w_1) . \tag{5.8}$$

Thus the vevs of the scalar fields are now written as

$$\begin{aligned} z_0 &= \cdots = z_{n-1} \equiv z \in \mathbf{C} , \\ w_1 &= \zeta \frac{z}{|z|} , \qquad\qquad w^2 = \zeta e^{i\phi} . \end{aligned}$$

So there is a surjection from the space $\mathbf{C} \times S^1$ onto the moduli space $\mathcal{M}$.

There is a residual $\mathbf{Z}_n$ symmetry whose generator acts on the scalar fields as

$$z_i \to e^{\frac{2\pi i}{n}} z_i , \qquad w_{1,2} \to e^{\frac{2\pi i}{n}} w_{1,2} . \tag{5.9}$$

They can be rewritten as follows,

$$\arg(z_i) \to \arg(z_i) + 2\pi \frac{1}{n} , \qquad \phi \to \phi + 2\pi \frac{1}{n} . \tag{5.10}$$

The action of elements of $\mathbf{Z}_n$ coincides with (5.6), up to an overall scale, so the moduli space

$$\mathcal{M} = \frac{(\mathbf{C} \times S^1)}{\mathbf{Z}_n} \tag{5.11}$$

is the KK-Melvin with $qR = 1/n$ modulo integer.

The sigma model metric can also be calculated. We parametrize the potential minima as

$$z_i = re^{i\theta_i}\,, \qquad w_1 = \zeta e^{i\phi_1}\,, \qquad w_2 = \zeta e^{i(\phi_2 + \frac{1}{n}\theta)}\,, \tag{5.12}$$

where $\theta = \sum_{i=0}^{n-1} \theta_i$. The phase of w_2 is shifted so as to take into account the twisted identifications (5.10). Then the kinetic terms for the scalars become

$$\sum_{i=0}^{n-2} \left\{ (\partial r)^2 + r^2 \left(\partial\theta_i - \left(A^i - A^{i+1}\right)\right)^2 \right\} + \left\{ (\partial r)^2 + r^2 \left(\partial\theta_{n-1} - \left(A^{n-1} - A^0\right)\right)^2 \right\} +$$
$$+ \zeta^2 (\partial\phi_1 - A)^2 + \zeta^2 \left(\partial\phi_2 + \frac{1}{n}\partial\theta - A \right)^2 .$$

The first two terms coincide with the kinetic terms in the worldvolume theory of the probe D-brane [8], so the gauge fields A^i can be integrated out in the same way. Another gauge field A can also be integrated out easily, and the result is

$$\frac{\zeta^2}{2} \left(\partial\phi + \frac{1}{n}\partial\theta \right)^2, \qquad \phi = \phi_2 - \phi_1\,. \tag{5.13}$$

Thus we obtain the following metric

$$ds^2 = ndr^2 + \frac{r^2}{n}d\theta^2 + \frac{\zeta^2}{2} \left(d\phi + \frac{1}{n}d\theta \right)^2, \tag{5.14}$$

from the resulting expression of the kinetic terms.

Let us show the relation between the metric (5.14) and that of the KK-Melvin. The metric can be rewritten as

$$ds^2 = n \left[dr^2 + \frac{r^2}{n^2}d\theta^2 + \left(\frac{\zeta}{\sqrt{2n}}d\phi + \frac{\zeta}{\sqrt{2n^3}}d\theta \right)^2 \right]. \tag{5.15}$$

Let $\zeta\phi/\sqrt{2} = ny$. The factor n is in front of y, since the radius of S^1 in the space $\mathbf{C} \times S^1$, which corresponds to the ϕ-direction, is n times larger than that of the y-direction in the KK-Melvin, as mentioned in the previous subsection. From this identification, the radius of the y-direction is

$$R = \frac{\zeta}{\sqrt{2}} n^{-\frac{3}{2}}\,. \tag{5.16}$$

Finally the metric becomes

$$ds^2 = n \left[dr^2 + \frac{r^2}{n^2}d\theta^2 + (ndy + Rd\theta)^2 \right]. \tag{5.17}$$

This exactly coincides, up to an overall scale, with the nontrivial part of the metric of the KK-Melvin in a coordinate system shown in [5], with $qR = 1/n$ modulo integer. Therefore we conclude that the linear sigma model we have discussed so far flows into the nonlinear sigma model of the KK-Melvin in the IR limit. Note that the radius R depends on n. This will become important later.

Since we have considered only bosonic degrees of freedom, one cannot distinguish the KK-Melvin with $qR = 1+1/n$ from that with $qR = 1/n$, within this model. The difference would become apparent when fermionic degrees of freedom are included in the model.

6. Deformations of the model

Now we discuss the closed string tachyon condensation in the KK-Melvin by analyzing effects of deformations of the potential of the linear sigma model.

Suppose that a tachyon vev with some profile is turned on. This would be described by a relevant deformation of the theory. Relevant deformations change the IR behavior of a theory. In a linear sigma model, the IR behavior is determined by the minima of the potential, so relevant deformations would correspond to perturbations of the potential which change the potential minima.

Naively, forms of such relevant deformations can be deduced from the following simplified argument. Assume a potential $V(x) = (x^2 - a^2)^2$. For simplicity, we assume that the system is symmetric under the sign flip of x; $x \to -x$. Then the generic perturbation of the potential is a polynomial $f(x^2)$. This polynomial can be rewritten as

$$f(x^2) = (x^2 - a^2)^2 g(x^2) + Ax^2 + B\,. \tag{6.1}$$

Thus this means that, in general, the only x^2 term is essential for the deformations of the potential minima. So this term would correspond to a relevant deformation of the theory.

Similar situation can be seen in the Landau-Ginzburg model. In this model, it is shown that adding, for example, the lowest order term corresponds to the most relevant deformation of a CFT.

Therefore it seems natural to expect that adding one of the lowest order terms to our linear sigma model would correspond to a relevant deformation which would describe turning on a tachyon vev.

We consider the following terms,

$$\sum_{i=0}^{n-1} m_i|z_i|^2 + \mu_1|w_1|^2 + \mu_2|w_2|^2\,, \tag{6.2}$$

with $\sum_{i=0}^{n-1} m_i = 0$. A reason for imposing this condition on m_i is as follows: The condition implies that we exclude the perturbation of the form $\sum |z_i|^2$. This perturbation lifts the moduli space and the dimension of the moduli space becomes smaller, even if the coefficient of the perturbation is small. We expect that the coefficients of the perturbations are related to the magnitude of the tachyon vev, and the effect of the tachyon condensation is continuous for the tachyon vev. So this perturbation would have nothing to do with the tachyon condensation.

The last two terms in (6.2) just change the potential of $w_{1,2}$ in (5.7) into the form

$$(|w_1|^2 - \zeta_1^2)^2 + (|w_2|^2 - \zeta_2^2)^2 + \text{const.} \tag{6.3}$$

The metric obtained from this deformed potential is essentially the same as (5.14). The difference is the value of ζ which is now determined by the equation

$$\frac{1}{\zeta^2} = \frac{1}{2}\left(\frac{1}{\zeta_1^2} + \frac{1}{\zeta_2^2}\right). \tag{6.4}$$

Therefore this perturbation only changes the radius R while keeping qR fixed. So this would rather describe the marginal deformation corresponding to the change of G_{yy}, which is discussed in section 3.2.

Let's focus on the other terms in (6.2). The effect of this perturbation to the metric is investigated in [8], and we can use the result in our case. The resulting metric is

$$ds^2 = n(r)dr^2 + \frac{r^2}{n(r)}d\theta^2 + \frac{\zeta^2}{2}\left(d\phi + \frac{1}{n}d\theta\right)^2. \tag{6.5}$$

We have used the same parametrization for $w_{1,2}$ as before. The function $n(r)$ behaves as

$$n(r) \to \begin{cases} n\,, & (r \to \infty) \\ n'\,, & (r = 0) \end{cases} \tag{6.6}$$

where n' is an integer satisfying $1 \leq n' < n$. The integer n' is determined from the values of m_i. The value of qR at $r = 0$ is thus different from the value at $r \to \infty$.

This result also indicates that the radius of the y-direction varies as r goes from zero to infinity,

$$R \to \begin{cases} \frac{\zeta}{\sqrt{2n^3}}\,, & (r \to \infty) \\ \frac{\zeta}{\sqrt{2n'n^2}}\,. & (r = 0) \end{cases} \tag{6.7}$$

Therefore this shows that $R(r \to \infty) < R(r = 0)$.

Recall that the closed string tachyons are localized around $r = 0$. Thus the geometry around $r = 0$ would be the one deformed by the tachyon condensation. So we could conclude that, after the tachyon condensation, the parameters qR, R become larger. The system with $qR = 2$ is the same theory as that with $qR = 0$. Therefore string theory on the KK-Melvin would decay into the theory on the flat spacetime times S^1, and supersymmetry would be restored. When the tachyon condensation is fine-tuned so that the endpoint of the decay is a tachyon-free KK-Melvin, the theory then continues to decay via an instanton effect [11].

The details of the above results we have obtained so far might not be correct. However, we expect that the qualitative picture is still meaningful. It is based on the following arguments: The ten-dimensional KK-Melvin can be lifted to the eleven-dimensional one, and it is related in [5] to a flat cone background in ten dimensions with a constant R-R field.

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If the arguments in [8] can be applied to this case, although a R-R background exists, one could claim that the scale of the radial direction is reduced, in addition to the change of the value of qR. So the relative scale of the y-direction would become larger, that is, the way of changing the parameters would match what we have found from the linear sigma model.

There are some pieces of evidence which support our results.

i. The tachyon condensation tends to drive the theory toward a tachyon-free theory, which can be seen from the figure 1.

ii. It is conjectured in [14] that type 0A theory is decompactified to be M-theory after the tachyon condensation. This is based on an argument on the nonzero cosmological constant in the theory. Since type 0A theory is conjectured to be M-theory compactified on S^1 with a twisted boundary condition, which is equivalent to M-theory compactified on the KK-Melvin, the above conjecture means that the KK-melvin becomes a trivial background and the radius of the S^1 becomes large, after the tachyon condensation. Our result is consistent with this conjecture.

iii. As pointed out in [7], the closed string tachyon condensation should induce the gauge symmetry breaking in heterotic theories. Consider a non-supersymmetric heterotic theory. This can be interpreted as a supersymmetric heterotic theory on the KK-Melvin with a Wilson line [7], unless the gauge group is E_8. If the radius of S^1 of the KK-Melvin becomes large while keeping the Wilson line fixed, then the gauge symmetry is broken by the Wilson line. Note that the tachyon condensation would also reduce the rank of the gauge group, not only break the gauge group to a subgroup with the same rank. We will comment on this point in the next section.

7. Tachyon condensation in heterotic theories

As has been pointed out, when closed string tachyons condense in heterotic theories, the gauge symmetry should be broken in general, since the tachyons couple to the gauge fields in a nontrivial way. Moreover, it is expected that the rank of the gauge group is reduced by the tachyon condensation. However, it is well-known that consistency conditions of the theory severely restrict the rank of the gauge group. Is it really possible for non-supersymmetric heterotic theories to decay into supersymmetric heterotic theories ?

Recall the spectra of the non-supersymmetric theories. There are seven such theories characterized by their gauge groups; the $\mathrm{SO}(16)\times\mathrm{SO}(16)$ theory is tachyon-free, the theories with $\mathrm{SO}(32)$, $E_8\times\mathrm{SO}(16)$, $\mathrm{SO}(8)\times\mathrm{SO}(24)$, $(E_7\times\mathrm{SU}(2))^2$, $\mathrm{U}(16)$, E_8 all have tachyons. First four theories have tachyons in nontrivial representations of the gauge groups.

Consider the $\mathrm{SO}(32)$ theory. The tachyons form the vector representation of the $\mathrm{SO}(32)$. Suppose that the tachyons acquire generic vevs. By a rigid gauge transformation, one can put them into the form,

$$T=\begin{pmatrix} * \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{7.1}$$

So the gauge group is broken from SO(32) to SO(31). Therefore the rank of the gauge group is reduced by one, due to the tachyon condensation. The similar reduction would occur in other theories. The reduction of the rank is at most one.

From the above considerations, one may think that the SO(32) theory would decay into the supersymmetric heterotic theory compactified on S^1, whose gauge group is SO(31), or its subgroup with the rank fifteen. However, such a heterotic theory does not known. The rank of the gauge group less than sixteen is at most eight [17]. So it may be concluded that the above non-supersymmetric heterotic theory would not be well-defined, since its stable vacuum does not seem to exist.

Similar situations are found in the supersymmetric heterotic theories on the KK-Melvin, even with a finite radius of the S^1. However, in this case, there would be a possible solution of this puzzle. Recall that the rank of the gauge group can be larger than sixteen at special points in the moduli space of compactified heterotic theories [18]. For example, the supersymmetric SO(32) theory on S^1 can have the gauge group SO(34) by choosing appropriate radius and Wilson line. Therefore it would be possible that the rank is sixteen even after the rank is reduced by one due to the tachyon condensation. The mechanism of this phenomenon, if exists, would be very complicated and the field theoretic interpretation would be difficult.

Of course there are other possibilities. One is that the endpoint of the decay is a CHL string [17], by further reducing the rank in some way. And one cannot exclude the possibility that the theory becomes a non-critical string. This might be preferred because of the c-theorem in CFT.

We expect that the first possibility would be realized. We think it natural because the theories we have considered are supersymmetric theories on a particular background. So the stable vacuum of such theories would probably be the supersymmetric vacuum.

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